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B.Sc II part Paper IV

Differential Equation (Singular Solution)

Let the diff. Eqn. be denoted by  $f(x, y, p) = 0$   
;  $p = \frac{dy}{dx}$  and its general solution by  $\phi(x, y, c) = 0$   
where  $c$  is an arbitrary constant. The existence  
of singular solution depends upon the  $p$ -discrimi-  
nant of  $f(x, y, p) = 0$  and the  $c$ -discriminant of  
 $\phi(x, y, c) = 0$ . Hence prior to embarking on the  
definition and existence of singular solution,  
we look at the discriminant of an equation  
whether ~~algebraic~~ algebraic or differential.

The discriminant of a quadratic eqn  
 $ax^2 + bx + c = 0$  is  $b^2 - 4ac$ . The vanishing  
of the discriminant i.e.  $b^2 - 4ac = 0$  express  
the condition that the two roots of the  
given eqn are equal. i.e. two values of  
 $x$  are equal. Similarly if  $f(x) = 0$  be any  
eqn. of higher order than two, then as we  
know from Theory of Equations that the  
condition for equal roots is obtained by  
eliminating  $x$  between  $f(x) = 0$  and  $f'(x) = 0$ .

Let  $f(x, y, p) = 0$  be a differential eqn. and let its  
solution be  $\phi(x, y, c) = 0$  where  $c$  is an arbitrary const.  
Then the  $c$ -discriminant is obtained by eliminating  
 $c$  between  $\phi(x, y, c) = 0$  and  $\frac{\partial \phi}{\partial c} = 0$ ; treating  $x, y$   
as constants and its vanishing expresses the

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Condition that the eqn.  $\phi(x, y, c) = 0$  regarded as an equation in  $c$ , has equal values  $c$ .

Hence we can say that the  $c$ -discriminant of  $\phi(x, y, c) = 0$  represents the locus for each point of which  $\phi(x, y, c) = 0$  has equal values of  $c$ .  
Again, if  $f(x, y, p) = 0$  be the differential equation whose general solution is  $\phi(x, y, c) = 0$ , then the  $p$ -discriminant is obtained by eliminating  $p$  between  $f(x, y, p) = 0$  and  $\frac{\partial f}{\partial p} = 0$ , treating  $x, y$  as constants and its vanishing expresses the condition that the equation  $f(x, y, p) = 0$  regarded as an equation in  $p$  has equal values of  $p$ .

Hence, we can say that the  $p$ -discriminant of  $f(x, y, p) = 0$  represents the locus for each point of which  $f(x, y, p) = 0$  has equal values of  $p$ .

$$\text{If } y = px + f(p); p = \frac{dy}{dx} \quad \text{--- (1)}$$

be a diff. eqn. in Clairaut's form, then its general solution is  $y = cx + f(c)$  --- (2)

where  $c$  is an arbitrary constant. Obviously the  $p$ -discriminant of (1) and  $c$ -discriminant of (2) are the same and that gives us what is called the singular solution of the D.E (1).

However, if  $f(x, y, p) = 0$  be <sup>(3)</sup> any diff. eqn. not necessary in Clairaut's form and if  $\phi(x, y, c) = 0$  --- (4) be its solution, then are we suppose that  $p$ -discriminant of (3) and  $c$ -discriminant of (4) are the same? obviously not.